

Belirsiz Integral

Tanım: $y=f(x)$ fonksiyonu verilsin. Eğer $f'(x)=f(x)$ ise $f(x)+c$ 'ye $y=f(x)$ fonksiyonunun belirsiz integrali denir ve

$$\int f(x) dx = f(x) + c \quad \text{şeklinde gösterilir. } c: \text{integral sabiti}$$

$$\int 1 dx = x + c \quad \int \cos x dx = \sin x + c$$

$$\int x dx = \frac{x^2}{2} + c \quad \int \frac{1}{x} dx, x > 0 = \ln x$$

Temel İntegrasyon Formülleri:

$$① \int 1 dx = \int dx = x + c$$

$$⑦ \int \sin x dx = -\cos x + c$$

$$② \int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$⑧ \int (1 + \tan^2 x) dx = \tan x + c$$

$$③ \int \frac{1}{x} dx = \int x^{-1} dx = \ln |x| + c$$

$$⑨ \int (1 + \cot^2 x) dx = -\cot x + c$$

$$④ \int e^x dx = e^x + c$$

$$⑩ \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$$

$$⑤ \int a^x dx = \frac{a^x}{\ln a} + c$$

$$⑪ \int \frac{-dx}{\sqrt{1-x^2}} = \arccos x + c$$

$$⑥ \int \cos x dx = \sin x + c$$

$$⑫ \int \frac{dx}{1+x^2} = \arctan x + c$$

$$⑬ \int \frac{-dx}{1+x^2} = \operatorname{Arccot} x + c$$

$$⑭ k \in \mathbb{R} \quad (k: \text{sabit})$$

$$\int k f(x) dx = k \int f(x) dx$$

$$⑮ \int [f(x) \mp g(x)] dx = \int f(x) dx \mp \int g(x) dx$$

$$\text{ör 1)} \int x^7 dx = \frac{x^8}{8} + C$$

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$$2) \int x^{-2} dx = \frac{x^{-1}}{-1} + C$$

$$3) \int 3 dx = 3x + C$$

$$4) \int \sqrt[3]{x} dx = \int x^{\frac{1}{3}} dx = \frac{7}{8} x^{\frac{8}{3}} + C$$

$$5) \int \frac{dx}{\sqrt{x}} = \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$6) \int \frac{2+3x}{x^3} dx = \int (2+3x) x^{-3} dx$$

$$7) \int (x^2+2x)^2 dx = \int (x^4+4x^3+4x^2)$$

$$= \int 2x^{-3} dx + \int 3x^{-2} dx = \frac{2x^{-2}}{-2} + \frac{3x^{-1}}{-1} + C$$

$$= -x^{-2} - 3x^{-1} + C$$

$$= \int x^4 dx + \int 4x^3 dx + \int 4x^2 dx = \dots$$

Değişken Değiştirme ile İntegrasyon Metodu

$$\text{ör 1)} \int x(x^2+2)^{300} dx = ?$$

$$x^2+2 = u \text{ dersek}$$

$$2x dx = du$$

$$u' = \frac{du}{dx}$$

$$x dx = \frac{du}{2}$$

$$\frac{du}{dx} = 2x$$

$$= \int u^{300} \frac{du}{2} = \frac{1}{2} \int u^{300} du$$

$$= \frac{1}{2} \cdot \frac{u^{301}}{301} + C = \frac{(x^2+2)^{301}}{602} + C$$

$$\text{ör 2)} \int x(2x+5)^{100} dx = ?$$

$$x = \frac{u-5}{2}$$

$$2x+5 = u$$

$$2 dx = du$$

$$dx = \frac{du}{2}$$

$$u' = \frac{du}{dx}$$

$$\Rightarrow \int \left(\frac{u-5}{2}\right) u^{100} \frac{du}{2} = \frac{1}{4} \int u^{101} - 5 \cdot u^{100} du$$

$$= \frac{1}{4} \frac{(2x+5)^{102}}{102} - \frac{5(2x+5)^{101}}{101}$$

$$\text{Bsp } \int x e^{x^2} dx = ?$$

$$x^2 = u$$

$$2x \cdot dx = du$$

$$x \cdot dx = \frac{du}{2}$$

$$= \int e^u \cdot \frac{du}{2} = \frac{1}{2} \int e^u \cdot du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

I. Brüche Rationale Funktionen integrieren

$$f(x) = \frac{p(x)}{q(x)}$$

$$= h(x) + \frac{r(x)}{q(x)}$$

$$q(x) = (x-c_1)(x-c_2) \dots (x-c_n)$$

$$\frac{r(x)}{(x-c_1)(x-c_2) \dots (x-c_n)} = \frac{A_1}{(x-c_1)} + \frac{A_2}{(x-c_2)} + \dots + \frac{A_n}{(x-c_n)}$$

$$\int f(x) dx = \int \left[h(x) + \frac{r(x)}{q(x)} \right] dx$$

$$\text{Bsp } \int \frac{dx}{4-x^2} = ?$$

$$\frac{1}{(2-x)(2+x)} = \frac{A}{(2-x)} + \frac{B}{(2+x)}$$

$$1 = A(2+x) + B(2-x)$$

$$1 = x(A-B) + 2(A+B)$$

$$0 = A-B$$

$$1 = 2(A+B)$$

$$A = B = \frac{1}{4}$$

$$\int \frac{1}{4} \frac{1}{(2-x)} dx + \int \frac{1}{4} \frac{1}{2+x} dx$$

$$= -\frac{1}{4} \ln|2-x| + \frac{1}{4} \ln|2+x| + C$$

$$\text{Bsp } \int \frac{x}{3x^2+8x-3} dx = ?$$

$$\frac{x}{(3x-1)(x+3)} = \frac{A}{(3x-1)} + \frac{B}{(x+3)}$$

$$3x^2+8x-3$$

$$\begin{array}{r} 3x \\ x \end{array}$$

$$\begin{array}{r} -1 \\ 3 \end{array} \quad (3x-1)(x+3)$$

$$x = A(x+3) + B(3x-1)$$

$$x = x(A+3B) + 3A-B$$

$$A+3B=1$$

$$3A-B=0$$

$$3A=B$$

$$A = \frac{1}{10}$$

$$B = \frac{3}{10}$$

$$\int \frac{1}{10} \frac{1}{(3x-1)} dx + \int \frac{3}{10} \frac{1}{x+3} dx$$

$$\frac{1}{10} \ln \left| \frac{3x-1}{3} \right| + \frac{3}{10} \ln|x+3|$$

Q $\int \frac{2x^3}{x^4+1} dx = ?$

$$\begin{aligned} x^4+1 &= u \\ 4x^3 \cdot dx &= du \\ dx &= \frac{du}{4x^3} \end{aligned} \quad = \int \frac{\cancel{2x^3} \cdot \frac{du}{\cancel{4x^3}}}{u} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln u = \frac{1}{2} \ln(x^4+1) + C$$

Q $\int \frac{dx}{x \ln x} = ?$

$$\begin{aligned} \ln x &= u \\ \frac{1}{x} dx &= du \end{aligned} \quad = \int \frac{du}{u} = \ln u + C = \ln(\ln x) + C$$

Q $\int e^{x^4} 4x^3 dx = ?$

$$\begin{aligned} x^4 &= u \\ 4x^3 \cdot dx &= du \end{aligned} \quad = \int e^u \cdot du = e^u + C = e^{x^4} + C$$

Q $\int \frac{dx}{1+e^x} = ?$

$$\begin{aligned} 1+e^x &= u \\ 1 \cdot \ln e \cdot e^x \cdot dx &= du \\ dx \cdot e^x &= du \end{aligned} \quad \begin{aligned} 1+e^x &= u \\ du &= e^x dx \end{aligned} \quad \frac{du}{e^x \cdot u} = \int \frac{du}{u(1+u)} \quad \ln(1+u)$$

Q $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx = ?$

$$\begin{aligned} e^x &= u \\ e^x \cdot dx &= du \end{aligned} \quad = \int \frac{du}{\sqrt{1-u^2}} = \text{ArcSin } u + C = \text{ArcSin } e^x + C$$